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INTERFACIAL CONDITIONS OF A CYLINDRICAL VORTEX SHEET.(U)

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Interfacial conditions for a cylindrical vortex sheet or a cylindrical fluid layer with radius-dependent density, velocity and magnetic fields are derived for isentropic compressible swirling flows subjected to arbitrary disturbances. Surface tension is included for possible immiscible fluids. These conditions are valid for both spatially and temporally growing waves and for flow profiles with or without discontinuities. The deformation of the sheet or layer affects the flow in two ways: perturbing the total pressure field and disturbing the centrifugal force field created by the azimuthal components of the velocity and the magnetic flux. It is shown, through

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20. ABSTRACTED (Continued)

the derivations of the interfacial conditions, that the Rayleigh-Synge and the Alfvén discriminant are in fact a differential representation of the centrifugal force field. Both discriminants play a crucial role in the stability characteristics of hydromagnetic swirling flows. As supported by a previously proposed mechanism, instabilities of such flows are both of centrifugal and of shear origin, the generalized Michael condition being a criterion for centrifugal stability. Centrifugal forces and velocity gradients affect flow characteristics through their interaction with the Lagrangian displacement. For centrifugally stable profiles, unstable waves with large wave numbers may be stabilized by centrifugal effects. This suggests a mechanism for the dilemma that the most unstable waves for flows of vortex sheet types correspond to the smallest-scale disturbances that are not observed in experiments.

CONTENTS

INTERFACIAL CONDITIONS OF A CYLINDRICAL VORTEX SHEET

1. INTRODUCTION

In a recent paper by Rotunno (1978), the uncertainties of the stability analysis for an incompressible cylindrical vortex sheet in a homogeneous inviscid fluid were again discussed. Based on a potential formulation for both vorticity-free regions inside and outside the vortex sheet, he resolved the inconsistency in an early analysis by Michalke & Timme (1967) and recovered the first two otherwise stable modes. However, some uncertainties of the stability analysis for flows with discontinuity profiles still remained unanswered. The purpose of this report is to present an overall view for flows of this type. The flow to be considered is isentropic and has general radius-dependent profiles for the density, velocity and magnetic fields. All dissipative effects are disregarded. Surface tension is also included for the case of immiscible fluids.

In an important paper on the stability of nondissipative swirling flows with and without magnetic influence, Howard & Gupta (1962) presented many interesting aspects on stability characteristics. Even though they restricted themselves mostly to homogeneous fluids subject to axisymmetric perturbations, their generalization of the Richardson criterion and of the semicircle theorem into axisymmetric steady flows presented a relatively simple insight into the type of parallel flows complicated by the cylindrical geometry. In their formulation of the problems, the Rayleigh criterion and the Michael condition played a dominant role in their analysis. Violation of those conditions results in no criteria. This is a crucial point. Fung & Kurzweg (1975) considered general types of inviscid swirling flows subject to arbitrary temporal disturbances. A sufficient condition for stability was derived and two interfacial conditions were obtained for possible discontinuity profiles existing in a transition fluid layer. According to their analysis, instability mechanisms for swirling flows have both centrifugal and shear origins, the

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Rayleigh-Synge criterion (Synge, 1933) being a condition for centrifugal stability. Violating the Rayleigh-Synge criterion automatically leads to violating the sufficiency condition. The Rayleigh-Synge criterion and the Michael condition were derived under the assumption that azimuthal disturbances and the axial components in the velocity and magnetic fields were suppressed. No radial or axial shear effects induced by the mean flow were allowed to upset flow stability. As a result, instabilities of this kind are simply the consequence of the unbalanced centrifugal forces created by the azimuthal components in the velocity and magnetic fields. Reminiscent of the stability encountered in two-dimensional stratified shear flows, the statically stable density distribution is a precondition for the stability analysis. Statically unstable stratified flows will automatically lead to the violation of the Richardson criterion. Instabilities of the Rayleigh-Taylor type will take place and the bound for growth rates will no longer be predicted by the semicircle theorem.

Even though the Richardson criterion and the semicircle theorem provide us with some upper bound information on stability or instability, criteria for flows of this kind do not yield sufficient knowledge of instabilities, if any, for a given flow profile. Solutions to the governing stability equations must be obtained before the detailed instability characteristics for the particular flow profile can be observed. Unfortunately, analytical solutions in terms of well-known functions for general swirling flows are very difficult to obtain except for a few broken-line profiles. Matching the solutions at the common boundary between two flow regions therefore becomes the trick for the analyses of this type. In matching those broken-line profiles, sufficient care for appropriate interfacial conditions must be taken. The cylindrical vortex sheet is one of the few profiles in which an exact solution to the governing stability equation exists. The particular velocity distribution examined by Michalke & Timme (1967) consists of a peripheral velocity jump which has a contribution to the centrifugal force balance at the vortex sheet. It will be shown, and as also pointed out by Fung & Kurzweg (1975), that the centrifugal forces arising from the perturbation at the interface should be taken into account. Furthermore, such perturbations will stabilize the lower modes with small wave numbers in vortex-sheet type profiles as the one considered by Michalke & Timme (1967).

In deriving the stability criteria for swirling flows, Howard & Gupta (1962) did not discuss cases in which discontinuities exist within the flow domain. In those cases, for which analytical solutions to the stability equations happen to be possible, instabilities are very likely to occur due to discontinuous configurations. The question which then arises is how those discontinuities fit into the criteria derived for continuous configurations. This report is meant to fill in the gap for stability characteristics between continuous and discontinuous configurations. We will present the interfacial conditions of a cylindrical vortex sheet or a cylindrical fluid layer for a general type of compressible swirling flows in the presence of a toroidal and an axial magnetic field. The perturbatic includes both spatially and temporally growing waves. Some insight into the stability characteristics may be observed from the interfacial conditions. It will be shown that the perturbation at the interface has a dual role: perturbing the total pressure field and disturbing the centrifugal force field created by the velocity and magnetic fields in the azimuthal direction. The latter has considerable influence on centrifugal stability or instability. It will be seen in the present study that the Rayleigh-Synge criterion and the generalized Michael condition [see Eq. (22)] are in fact the differential balance representations of a stable centrifugal force field. For centrifugally stable configurations, i.e., flows in which the Rayleigh-Synge criterion or the generalized Michael condition is satisfied, unstable waves with large wave numbers will be stabilized by centrifugal effects. This gives an explanation for the dilemma that the most unstable waves for flows of vortex sheet types correspond to the smallest-scale disturbances that are not observed in experiments.

2. GOVERNING EQUATIONS FOR NORMAL MODES

Consider a swirling flow with a velocity $\overline{U}$ to be confined within the annular region (r,θ,z) between two rigid, infinite and coaxial cylinders in the presence of a magnetic field $\overline{H}$. The fluid having an inhomogeneous density ρ is assumed to be compressible but non-heat-conducting. In the absence of gravitational forces and dissipation effects due to viscosity, magnetic resistivity and thermal diffusivity, the governing equations for the isentropic motion of the flow are

$$\rho \frac{D\bar{U}}{Dt} = \nabla Q + \frac{\mu}{4\pi} (\bar{H} \cdot \nabla) \bar{H} \tag{1}$$

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \bar{U} = 0 \tag{2}$$

$$\frac{\partial \bar{H}}{\partial t} = \nabla \times (\bar{U} \times \bar{H}) \tag{3}$$

$$\nabla \cdot \bar{H} = 0 \tag{4}$$

$$\frac{DP}{Dt} = a^2 \frac{D\rho}{Dt} \tag{5}$$

where μ denotes the magnetic permeability. The total pressure Q (including the magnetic pressure) is related to the hydrodynamic pressure P as follows:

$$Q = P + \frac{\mu}{4\pi} |\bar{H}|^2. \tag{6}$$

The velocity of sound for an isentropic process is given by

$$a^2 = \left(\frac{\partial P}{\partial \rho} \right)_{\text{isentropic}}.$$

For given velocity and magnetic fields only one of the thermodynamic variables can be independently prescribed. The boundary conditions for the system governed by Eqs. (1) to (5) are those of perfectly conducting rigid walls.

The flow to be considered has a steady-state, radius-dependent profile: $V_0(r)$ is the azimuthal velocity, $W_0(r)$ the axial velocity, $H_\theta(r)$ the azimuthal magnetic field, $H_z(r)$ the axial magnetic field, $Q_0(r)$ the total pressure, $\rho_0(r)$ the density and $a_0(r)$ the sound speed. Let the flow be perturbed as follows:

$$\begin{aligned} \bar{U} &= \bar{U}\,[\hat{u},\ V_0(r) + \hat{v},\ W_0(r) + \hat{w}], \\ \bar{H} &= \bar{H}\,[\hat{h}_r,\ H_\theta(r) + \hat{h}_\theta,\ H_z(r) + \hat{h}_z], \\ Q &= Q_0(r) + \hat{q}, \\ \rho &= \rho_0(r) + \hat{\rho}, \\ a &= a_0(r) + \hat{a}. \end{aligned} \tag{7}$$

We further introduce the periodic solutions

$$\hat{\phi} = \phi(r)\ Exp\ [i(kz + m\theta - \omega t)] \tag{8}$$

such that the azimuthal wave number m is an integer, and the axial wave number k and the circular frequency ω are both complex in order to admit solutions for both spatially and temporally growing disturbances. Within the frame work of the normal mode approach, the linearized equations for the flow described by Eqs. (1) to (5), subject to small perturbations, are given as follows:

$$\rho_0[i N u - \frac{2V_0}{r} v] - \frac{\mu}{4\pi}[i N_a h_r - \frac{2H_\theta}{r} h_\theta] - \frac{v_0^2}{r}\rho = -D q - i\frac{T}{R}(1-m^2-k^2R^2)\left\{\frac{u}{N}\right\}\delta(r-R) \quad (9)$$

$$\rho_0[i N v + D^*(V_0)u] - \frac{\mu}{4\pi}[i N_a h_\theta + D^*(H_\theta)h_r] = -i\frac{m}{r} q \quad (10)$$

$$\rho_0[i N w + (DW_0) u] - \frac{\mu}{4\pi}[i N_a h_z + (DH_z)h_r] = -i k q \quad (11)$$

$$i N\rho + (D\rho_0)u + \rho_0\left[D^*u + i\left\{\frac{m}{r}v + kw\right\}\right] = 0 \quad (12)$$

$$N h_r - N_a u = 0 \quad (13)$$

$$i N h_\theta - D_*(V_0)h_r - [i N_a v - D_*(H_\theta)u] = -H_\theta\left[D^*u + i\left\{\frac{m}{r}v + kw\right\}\right] \quad (14)$$

$$i N h_z - (DW_0) h_r - [i N_a w - (DH_z)u] = -H_z\left[D^*u + i\left\{\frac{m}{r}v + kw\right\}\right] \quad (15)$$

$$D^*h_r + i\left\{\frac{m}{r} h_\theta + kh_z\right\} = 0 \quad (16)$$

$$i N q + \left\{\rho_0\frac{V_0^2}{r} - \frac{\mu}{4\pi}\left[\frac{H_\theta^2}{r} + D\left\{\frac{H_\theta^2+H_z^2}{2}\right\}\right]\right\} u - iN\frac{\mu}{4\pi}(H_\theta h_\theta + H_z h_z)$$
$$= a_0^2[i N \rho + (D\rho_0)u] \quad (17)$$

where $N = kW_0 + m\frac{V_0}{r} - \omega$ is the Doppler-shifted frequency, $N_a = kH_z + m\frac{H_\theta}{r}$, $D = \frac{d}{dr}$, $D^* = D + \frac{1}{r}$ and $D_* = D - \frac{1}{r}$. The surface tension effect for possible immiscible fluids is introduced in Eq. (9), with T representing the surface tension coefficient, $\delta(r - R)$ the Dirac delta function, and R the radial position for a cylindrical vortex or a cylindrical fluid layer. The characteristic that Eqs. (3) and (4) under the present assumption represent only three independent partial differential equations is reflected in Eqs. (13) to (16).

When the azimuthal and axial Alfvén velocities

$$V_A = \sqrt{\frac{\mu}{4\pi\rho_0}} H_\theta, \quad W_A = \sqrt{\frac{\mu}{4\pi\rho_0}} H_z$$

and the Alfvén frequency

$$N_A = k\ W_A + m\frac{V_A}{r}$$

are defined as shown, Eqs. (9) to (17) can be combined into two independent differential equations as follows:

$$\left(1 - \frac{N_A^2}{N^2}\right) D^* \left(\frac{u}{N}\right) - \frac{2mV_0}{N\,r^2}\left(1 - \frac{N_A}{N}\,\frac{V_A}{V_0}\right)\left(\frac{u}{N}\right) - \frac{i}{\rho_0}\,\frac{k^2 + \frac{m^2}{r^2}}{N^2}\,q = -\,\frac{G}{a_0^2} \tag{18}$$

$$\left\{\left(1 - \frac{N_A^2}{N^2}\right)\left[(N^2 - \Phi) - (N_A^2 - \Psi_A) + \frac{T}{\rho_0 R^2}(1 - m^2 - k^2R^2)\delta(r - R)\right]\right.$$

$$\left. - 4\,\frac{V_0^2}{r^2}\,\frac{N_A^2}{N^2}\left[\left(1 - \frac{V_A}{V_0}\right)^2 + 2\,\frac{V_A}{V_0}\left(1 - \frac{N}{N_A}\right)\right]\right\}\left(\frac{u}{N}\right)$$

$$-\,\frac{i}{\rho_0}\left[\left(1 - \frac{N_A^2}{N^2}\right)Dq + \frac{2mV_0}{Nr^2}\left(1 - \frac{V_A}{V_0}\,\frac{N_A}{N}\right)q\right] = \frac{V_0^2}{r}\left[\left(\frac{N_A}{N} - \frac{V_A}{V_0}\right)^2 - \left(1 - \frac{V_A^2}{V_0^2}\right)\right]\frac{G}{a_0^2} \tag{19}$$

where

$$G = (V_A^2 + W_A^2)D^*\left(\frac{u}{N}\right) - \frac{N^2}{N^2 - N_A^2}\left\{\frac{V_0^2}{r}\left[\left(\frac{N_A}{N} - \frac{V_A}{V_0}\right)^2 - \left(1 - \frac{V_A^2}{V_0^2}\right)\right]\right.$$

$$\left. + \frac{V_A^2 + W_A^2}{r}\left[\frac{2mV_0}{N\,r}\left(1 - \frac{N_A}{N}\,\frac{V_A}{V_0}\right)\right]\right\}\left(\frac{u}{N}\right)$$

$$+\,\frac{i}{\rho_0}\,\frac{N^2}{(N^2 - N_A^2)}\left[1 - (V_A^2 + W_A^2)\frac{k^2 + \frac{m^2}{r^2}}{N^2}\right]q$$

The Rayleigh-Synge discriminant is defined as

$$\Phi = \frac{D[\rho_0(r\ V_0)^2]}{\rho_0 r^3} \tag{20}$$

while the Alfvén discriminant is defined as

$$\Psi_A = \frac{r}{\rho_0}\,D\left[\rho_0\left(\frac{V_A}{r}\right)^2\right] \tag{21}$$

These two discriminants play a crucial role in the flow stability. For incompressible flows subject to axisymmetric disturbances and with all the axial influence in the velocity and magnetic field suppressed, they constitute a necessary and sufficient condition for stability, i.e.,

$$\Phi - \Psi_A \geqslant 0. \tag{22}$$

The above criterion can be easily obtained from Eqs. (18) and (19), and will be called the generalized Michael condition. Rayleigh and von Karman (see Lin, 1955) respectively presented the physical explanation for the Rayleigh criterion based on the kinetic energy argument and on the centrifugal force balance. There is no difficulty in extending their arguments to heterogeneous fluids. The same arguments can be used to explain the role played by the Alfvén discriminant in stability except that the key point is the conservation of angular magnetic flux rather than the conservation of angular circulation. The influence of these two discriminants on stability will be further discussed later.

Assuming a vortex sheet or a fluid layer located at $r = R$ with possible discontinuities in all components of the steady state density, velocity and magnetic fields, one can integrate Eqs. (18) and (19) across the vortex sheet to obtain the kinematic and dynamic interfacial conditions:

$$<\frac{u}{N}> = 0 = <\frac{h_r}{N_A}> \tag{23}$$

$$<q> - i\left(\frac{u}{N}\right)_R \left[<\rho_0\left(\frac{V_0^2}{r} - \frac{V_A^2}{r}\right)> + \frac{T}{R^2}\,(k^2R^2 + m^2 - 1)\right] = 0, \tag{24}$$

where $<\phi> = \phi(r_{+o}) - \phi(R_{-o})$ denotes a possible jump condition at the interface. Equation (23) simply states the well-known fact that the Lagrangian displacement should be continuous across the interface. Equation (24) points out that the dynamic interfacial condition should include not only the perturbations from the surface tension and the total pressure field, but also the perturbations from the centrifugal effects resulting from the velocity and Alfvén waves in the azimuthal direction. It should be pointed out that the jump condition arising from the latter perturbations is the outcome of integrating the Rayleigh-Synge and the Alfvén discriminant across the interface. This is an important point for the stability characteristics to be discussed later. Compressibility effects do not explicitly enter into either of the interfacial conditions. This characteristic can be seen from an alternative derivation of both conditions in the following section.

3. PHYSICAL CONSIDERATIONS

Equations (23) and (24) can also be derived through a simple linear perturbation method. Assume that the cylindrical vortex sheet located at the radial position R is disturbed such that the deformed interface is prescribed by

$$r = R + \hat{\eta}(r,\theta,z;t) \tag{25}$$

where $R >> \hat{\eta}$. Taking the total derivative of Eq. (25) and assuming periodic solutions yield

$$\eta(r) = -i\frac{u(r)}{kW_0 + m\frac{V_0}{r} - \omega}. \tag{26}$$

Equation (23) follows if no gap is allowed to exist at the disturbed interface. Since both η and u are small compared with the quantities of the steady flow, Eq. (26) also suggests that larger wave numbers correspond to smaller displacements. As will be seen in the next section, this well-known behavior will provide us with some insight into both temporal and spatial instabilities without detailed investigations into solutions or criteria for the governing stability equations.

The dynamic interfacial condition can also be obtained by examining the Euler equation of motion. In the presence of surface tension effects, the steady-state form of Eq. (1) in the radial direction yields

$$DQ_0 = \rho_0\left(\frac{V_0^2}{r} - \frac{V_A^2}{r}\right) - \frac{T}{R}\,\delta(r - R). \tag{27}$$

The steady-state total pressures inside and outside the vortex sheet are respectively

$$Q_{01}(r) = \int_{R_1}^{r} \rho_{01}\left(\frac{V_{01}^2}{\xi} - \frac{V_{A1}^2}{\xi}\right) d\xi \qquad \text{for } R_1 \leqslant r \leqslant R \tag{28a}$$

$$Q_{02}(r) = \int_{R_1}^{R} \rho_{01}\left(\frac{V_{01}^2}{\xi} - \frac{V_{A1}^2}{\xi}\right) d\xi + \int_{R}^{r} \rho_{02}\left(\frac{V_{02}^2}{\xi} - \frac{V_{A2}^2}{\xi}\right) d\xi - \frac{T}{R} \tag{28b}$$

$$\text{for } R \leqslant r < \infty,$$

where the subscripts 1 and 2 respectively denote the quantities prescribed in the inner and outer regions separated by the vortex sheet, and R_1 is a reference radial location. Let the vortex sheet be perturbed according to Eq. (25). The total pressure should be balanced at the perturbed interface, i.e.,

$$Q_1 (R + \hat{\eta}) = Q_2 (R + \hat{\eta}) + T \left[\frac{1}{R_{r\theta}} + \frac{1}{R_{rz}} \right] \tag{29}$$

where $R_{r\theta}$ and R_{rz} are the principal curvatures of the disturbed surface. Subtracting Eqs. (28) from Eq. (29) and assuming that all the quantities in the mean flow are bounded and continuous in the interval $[R, R + \hat{\eta}]$, we obtain the first order perturbation condition for dynamical balance evaluated at the undeformed interface as follows:

$$<\hat{q}> + \left[<\rho_0 \left(\frac{V_\theta^2}{r} - \frac{V_A^2}{r} \right) > + T \left[\frac{\partial^2}{\partial z^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2} \right] \right] \hat{\eta}(R) = 0. \tag{30}$$

The dynamic interfacial condition (24) derived by the normal mode analysis follows if periodic solutions for the perturbation quantities are assumed once again.

As shown in Fig. (1), the mathematical steps adopted to derive Eq. (30) from Eq. (29) simply demonstrate a dissolution of the total pressure force acting at the *disturbed* surface of the vortex sheet (Fig. 1a) into the individual force components acting at the *steady-state* interface (Fig. 1b). As a matter of fact, Eq. (30) also can be reached simply by balancing all the force components acting at a differential element $\hat{\eta}_{(R)} R \, d\theta$ (per unit axial wave length) that experiences the centripetal acceleration induced by the angular velocity and magnetic flux. This procedure of force decomposition clearly demonstrates that the deformation of the interface described by Eq. (25) induces the perturbation to the flow in two ways: the perturbation to the pressure field (including the magnetic pressure) and the perturbation to the centrifugal force field arising from the azimuthal velocity and magnetic flux. Any discontinuity arising in densities, azimuthal velocity and magnetic fields should be included in the jump condition given by Eq. (24) or (30).

The derivation given here clearly demonstrates that Eqs. (26) and (30) are, respectively, pure kinematic and dynamic conditions. They are therefore valid for both compressible and incompressible flows, and as a result, compressibility effects do not explicitly enter conditions (23) and (24) derived in Section 2.

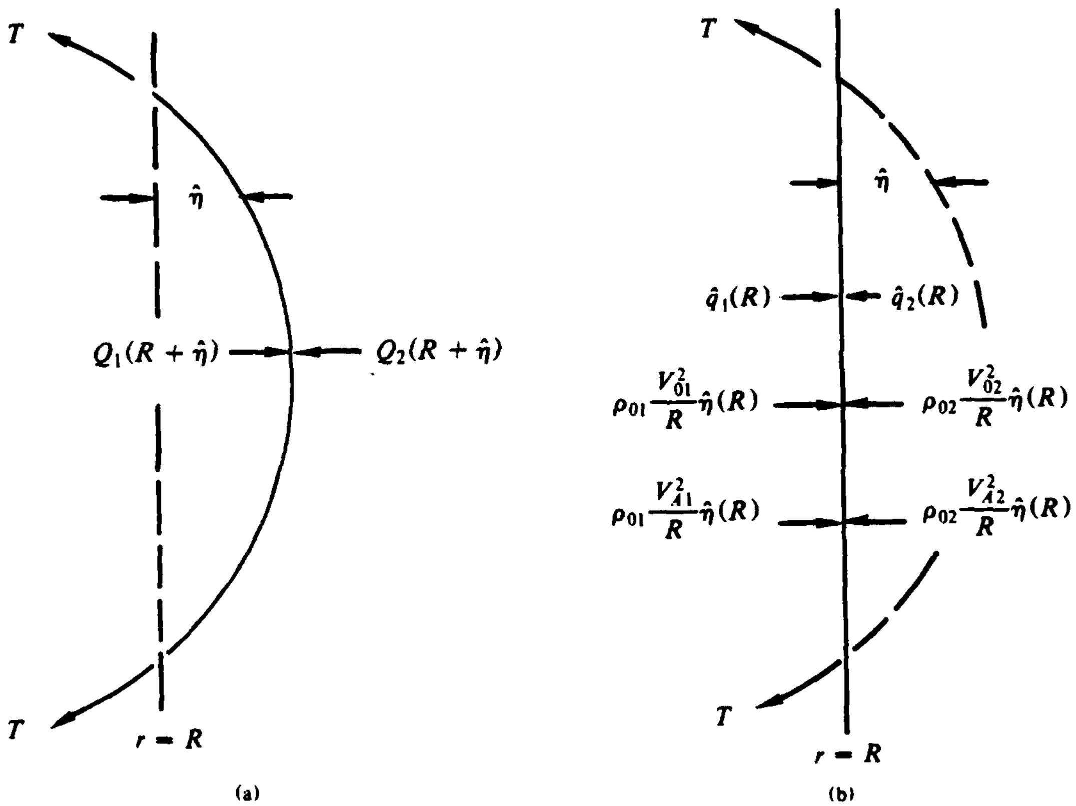

Figure 1 – Dissolution of the total force into the force components at the interface

4. SOME STABILITY CHARACTERISTICS

While many stability criteria for waves growing in time have been developed for various parallel flows within the framework of the Miles-Howard theorem, no general stability criteria known to the author have been established for waves growing in space. Realistically speaking, spatially growing waves dominate most of the characteristics for flow instabilities. The difficulty in establishing a stability criterion for spatial growth stems from the involvement of spatial derivatives in the Euler and Maxwell equations. While spatial stability criteria for general swirling flows remain unattainable, we will try to obtain some insight into the onset of instability for both spatially and temporally growing waves by examining the interfacial conditions derived in Sections 2 and 3.

It was pointed out by Fung & Kurzweg (1975) in their temporal stability analysis that the instability mechanisms for swirling flows are of both centrifugal and of shear origin, the classical Rayleigh-Synge criterion being a condition for centrifugal stability. Although the axial velocity always tends to destabilize the flow, the angular velocity plays a dual role in swirling instabilities. The centrifugal force produced by the rotation of fluids may have stabilizing effects, while the angular shear created by the azimuthal velocity gradients always has destabilizing results. The Rayleigh-Synge and the generalized Michael criteria were derived under the assumption that perturbations were axisymmetric and all axial components of the mean flow were suppressed. In general, it is expected that the growing disturbances, if any, will first amplify along the directions of the mean flow and of the magnetic flux. Disturbances of this kind, however, were restrained in the formulation of the two criteria. It therefore can be concluded that instabilities of this kind are not caused by shear effects but rather result from the unbalanced centrifugal forces that are created by the azimuthal component in the velocity and the magnetic field. This characteristic can be seen clearly from the physical explanation for the two criteria based on the discussion of kinetic energy and centrifugal forces. Results from disturbing such conservative systems are independent of the path of perturbations, in time or in space.

The Rayleigh-Synge and the Alfvén discriminants play a crucial role in the general stability characteristics of hydromagnetic swirling flows subject to arbitrary spatial and temporal disturbances. The Rayleigh-Synge criterion or the generalized Michael condition will have to be satisfied before a sufficient condition for stability or a semicircle theorem for unstable waves becomes valid. Violations of the two criteria result in a state of centrifugal instabilities. In our discussion of stability characteristics to be given, we will confine ourselves to those centrifugally stable configurations where the generalized Michael condition is satisfied.

Even though the interfacial conditions were derived for a cylindrical vortex sheet, they are valid for any finite transition layer with or without discontinuities provided that the thickness of the layer is small, i.e., $\hat{\eta} << R$. Several stability characteristics may be observed from the two conditions. Let us

consider a transition layer with thickness $\hat{\eta}$ located at a mean radial position $r = R$. For discussion convenience, the dynamic interfacial condition is rewritten as follows:

$$< \hat{q} > + \left[< \rho_o \left(\frac{V_o^2}{r} - \frac{V_A^2}{r} \right) > + \frac{T}{R^2} (k^2 R^2 + m^2 - 1) \right] \hat{\eta}(R) = 0 \tag{31}$$

the quantities within the square bracket in Eq. (31) represent the force components acting at both sides of the layer. Furthermore, it will be seen, by recapitulating some of the classical stability criteria for rotating flows, that these quantities have stabilizing or destabilizing effects depending on whether the sign of their sum is positive or negative.

The last term inside the square bracket represents the surface tension influence on the flow behavior by immiscible fluids. It is well known that surface tension always stabilizes the flow except for axisymmetric disturbances with long axial wave lengths.

The second term inside the square bracket describes a balance condition of the centrifugal force field created by the rotation and the azimuthal magnetic flux of the flow. This jump condition, which results from integrating the Rayleigh-Synge and the Alfvén discriminants across the interface as given in section 2 and from decomposing the total forces acting at the disturbed fluid layer as described in section 3, is clearly independent of the path of perturbation, in time or in space. Stability characteristics deduced from the behavior of this term are therefore valid for both spatially and temporally increasing waves. Furthermore, since this jump condition is an outcome of the integration of the Rayleigh-Synge and the Alfvén discriminant, a positive jump is in fact an integral representation of the generalized Michael condition, stabilizing the flow. Such stabilization is expressed in terms of the centripetal acceleration experienced by a differential element $\hat{\eta}\ Rd\theta$ per unit axial wave length as shown in Fig. 1b. In other words, the magnitude of such stabilization for a centrifugally stable profile is proportional to the thickness of a transition layer. Let us also point out that the thickness of a transition layer described by Eq. (26) depends inversely on the Doppler-shifted or the Alfvén frequency. Although the Doppler-shifted frequency N can be any value, depending on the wave number to be considered, the radial velocity perturbation $u(r)$ must be small in order to have a small value of $\eta(r)$ confined within

the normal mode analysis (the behaviors at the critical layer are not within the scope of the present discussion). For a centrifugally stable profile, the shear layer thickness plays a predominant role in flow instabilities of the Kelvin-Helmholtz type through its interactions with the velocity and the magnetic field in two ways. For a given flow configuration on both sides of the shear layer, a smaller thickness produces less centrifugal stabilizing effects as governed by the first term inside the square bracket in (30), and at the same time generates sharper velocity gradients, both destabilizing the flow.

The usual dilemma of stability analyses for a vortex sheet is that the most-amplified waves correspond to the smallest-scale of disturbances. One may resolve this problem to some extent by assuming that these smallest scale disturbances are quickly damped by a diffusion process. We would like, however, to present a mechanism for these phenomena based on the properties that stabilizing effects produced by the centrifugal force field and that shear effects created by the velocity gradients are both conveyed by the transition layer thickness. Vortex sheets are most unstable for large wave numbers which correspond to very thin shear layers, according to Eq. (26). Instability will first develop at the sheet due to the strong shear effect and small centrifugal stabilizing influence. Instability, however, will inevitably lead to the growth of the shear layer. A thicker layer, determined by a smaller wave number, will quickly stabilize the flow by conveying smaller velocity gradient and larger centrifugal stabilization influences, according to the previous discussion. Instability waves for lower wave numbers will immediately come into the picture until the final stage of linear instability has reached. This process for the evolution of stability modes was clearly demonstrated by Weske & Rankin (1963, Fig. 5) in their experimental investigation of cylindrical vortex motions subject to the restraints of the peripheral periodicity.

For a cylindrical vortex sheet with a non-rotating core, such as the one considered by Michalke & Timme (1967), the azimuthal velocity jump at the sheet does result in a positive quantity within the square bracket in Eq. (31), corresponding to a centrifugal influence that should have stabilized the first two less unstable modes. Examples in which perturbations with larger wave numbers are more stable

than those with smaller wave numbers can also be found for some particular profiles treated by Fung & Kurzweg (1973, 1975).

As suggested by Eq. (31), the thickness of a transition layer plays a dominant role in both centrifugal and shear instabilities. Short waves for centrifugally stable profiles are expected to amplify faster in thin transition layers. It is especially true when a component of the spatial perturbations is suppressed, i.e., either k or m is equal to zero. When general swirling flows are considered, three dimensional perturbations do come into the picture. As suggested by Eq. (26), the early stage for the onset of instability will take place in the neighborhood where the Lagrangian displacement is minimal (corresponding to a maximal Doppler-shifted frequency if no singularity is present). However, instability modes with large wave numbers will quickly be stabilized by centrifugal forces. Successive modes with smaller wave numbers, which are observed in experiments and are shown by analyses of flows with finite transition layer, will take place at a point where the wave lengths are of the same order of the layer thickness.

5. CONCLUSIONS

The interfacial conditions for a cylindrical vortex sheet or fluid layer with radius-dependent density, velocity and magnetic fields have been obtained for isentropic compressible flows subjected to arbitrary spatial and temporal disturbances. The disturbances affect the flow field in two ways: perturbing the total pressure field and disturbing the centrifugal force field created by the fluid rotation and the azimuthal magnetic field. While the influence of the former perturbation will not be observed without explicit solutions to a prescribed flow profile, the influence of the latter perturbation may lead to some insights into stability characteristics without explicitly solving the governing equations.

The first order perturbation of the centrifugal force field can be obtained either by integrating the Rayleigh-Synge and the Alfvén discriminant across the transition layer or by disturbing the centrifugal force field created by the azimuthal components of the velocity and magnetic fields. Based on this

argument, it is therefore concluded that the Rayleigh-Synge and the generalized Michael conditions, which apply to axisymmetric disturbances with the axial velocity and the axial magnetic flux suppressed, are in fact the differential representations of a stable centrifugal force field. The generalized Michael condition is a criterion for centrifugal stability. It is shown, from all the general stability criteria derived so far based on the Miles-Howard theorem, that violations of the axisymmetric condition without axial flow gradients automatically lead to violations of the general stability conditions. This characteristic implies that for a centrifugally unstable profile, instabilities of the Rayleigh-Taylor type will occur naturally.

The centrifugal force (or its gradient) influences the flow stability through its interaction with the Lagrangian displacement (or transition layer) that is inversely proportional to the wave numbers. The thickness of the transition layers approaches zero as the wave number approaches to infinity, a case for vortex sheet type profiles. *Flow profiles of this type are most susceptible to instabilities for the smallest* disturbances. This characteristic stems from the fact that the smallest disturbances convey the strongest shear and the least centrifugal stabilizing effects at the transition layer. As the onset of instability develops, displacements with smaller wave numbers will quickly appear. This development, in addition to reducing shear effects at the transition layer, will lead to the increase of centrifugal effects that stabilize the smallest disturbances. In consequence, the resultant instabilities observed in experiments are those corresponding to longer wave lengths.

Stability characteristics for swirling flows have important applications to many practical problems. For example, blowing cold air into the cores of a trailing vortex pair and injecting salinity into vortices in a Kármán vortex street can produce negative density gradients that will speed up the vortex breakdown process through the mechanism of centrifugal instabilities. Very fast rotation with minimal axial and azimuthal velocity gradients in counterflow centrifuges will produce a strong and stable density stratification required for separation of uranium isotopes. Strong negative magnetic gradients in the azimuthal direction with or without rotation of the flow can create a stable magnetic confinement for

laser fusion products during a controlled thermonuclear process. While it is extremely difficult to obtain stability criteria for the general compressible swirling flow considered here (especially for spatially growing waves), the present analysis provides us with some insight into the stability characteristics without explicitly solving the governing stability equations. Perhaps, it may lead to development of some more stringent criteria in the future.

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